

The entropy of the experimental dictionary is 8.884 binary digits per dictionary entry. The average length of the 5,208 codes in the dictionary encoding is 8.920 binary digits per entry. Defining the efficiency η of an encoding as

$$\eta = \frac{-\sum p_i \log_2 p_i}{\sum p_i L_i},$$

the efficiency of the dictionary encoding is $8.884/8.920 = 0.996$.

Code lengths ranged from 4 to 17 binary digits in the following distribution:

Binary Digits	Codes	Binary Digits	Codes
1	0	10	91
2	0	11	134
3	0	12	278
4	3	13	519
5	1	14	908
6	6	15	1241
7	14	16	1647
8	24	17	302
9	40		

A total of 75,983 binary digits were required for the complete encoding.

The encoding for the letter mode averaged 4.09 binary digits per letter as compared with a computed entropy of 4.03 binary digits per letter.

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Computer-Made Perspective Movies as a Scientific and Communication Tool*

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It is easy to program the basic transformation required for a perspective drawing. This fact plus the advent of high speed microfilm printers such as the General Dynamics Electronics S-C 4020 makes possible perspective movies as the direct output from a computer. The programming of such a movie is briefly described for studying the angular motions of a satellite containing an attitude control system. In the movie, a domino-shaped box represents the satellite and a sphere with circles of latitude and longitude represents the earth. The cost was approximately three to eight minutes of IBM 7090 time per one minute of movie.

Introduction

The basic transformation needed to make a perspective drawing is mathematically trivial to state. This fact along with the advent of plotting devices means that perspective drawings can be completely produced by

machine. Furthermore, a microfilm printer such as the General Dynamics Electronics S-C 4020 plots rapidly and directly on a sequence of film frames. Thus as the direct output of the microfilm printer, one may obtain a movie, ready for immediate viewing and with each frame drawn in perspective to render a three-dimensional representation. First of all, this offers a useful scientific tool, as one now has two additional coordinates, one spatial and the other time, for representing data. More important, it gives the ability to see a process evolve in time. Secondly, a movie, and particularly a perspective movie, is an important way of communicating results. Instead of the usual written report, one can even conceive of the computer making a self-contained motion picture for conveying research findings. This may have far greater impact than a verbal description.

These ideas are illustrated in a movie¹ made for a study of attitude control of a satellite. The attitude control system for this satellite consisted of the action of two single-axis gyros and gravity-gradient torques. From the movie, one can easily follow the motion of the satellite and the control system. This is not easily done from the inspection of plots of satellite angles versus time.

* Received October, 1963.

¹ A 16-mm copy of the movie may be obtained on loan from: Technical Information Library, Bell Telephone Laboratories, Inc., Murray Hill, N. J.

Perspective Drawing Subroutine

Let VP be the vantage point (Figure 1) of a perspective drawing of the object B , and \mathbf{R} a vector from VP to a point on B . A perspective drawing consists of points formed by vectors \mathbf{R} piercing a picture plane. If \mathbf{U} is a unit vector from VP normal to the picture plane and L

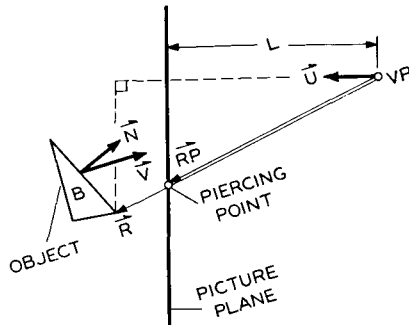


FIG. 1. Geometry entering into the computation of a perspective drawing

is the distance to the picture plane along \mathbf{U} , the vector \mathbf{RP} from VP to the piercing point of \mathbf{R} is simply

$$\mathbf{RP} = \frac{L}{(\mathbf{R} \cdot \mathbf{U})} \mathbf{R}. \quad (1)$$

Here $\mathbf{R} \cdot \mathbf{U}$ is the dot product of \mathbf{R} and \mathbf{U} . A subroutine to compute the right side of Eq. (1) is the heart of a perspective-drawing program.

Application to Attitude Control Studies

In the computer-made movies, the satellite is represented as a domino-shaped box, with plus signs used to identify the various sides. Also, the spin vectors of the gyros and the gyro stops are projected onto two sides of the box. This enables the viewer to follow the action of the gyros as well as that of the satellite proper.

At most, three sides of the box are seen in one given view. To test which sides, one computes the angle between an exterior normal \mathbf{N} to a side and a vector \mathbf{V} to the vantage point (Figure 1). If this angle is less than 90° , the side is seen; otherwise, it is not seen or appears edge on.

An idea of what the movie looks like is shown in Figure 2. The closely-spaced drawings of the domino box, in the style of rapid-sequence photography, show the satellite completing one orbit around a stationary earth. In the movie, the earth turns as the satellite goes around.

The movie is divided into two parts. The first part shows the motion of the satellite when it is given an initial yaw rate of four times the orbital rate. Three different views of this motion are given to illustrate the pictorial possibilities of computer-made movies. Two views, from different vantage points, show the satellite in orbit around a rotating earth. A third view shows the satellite in the orbiting reference frame, thus illustrating the gyro action and the bistability of so-called gravity-gradient systems.

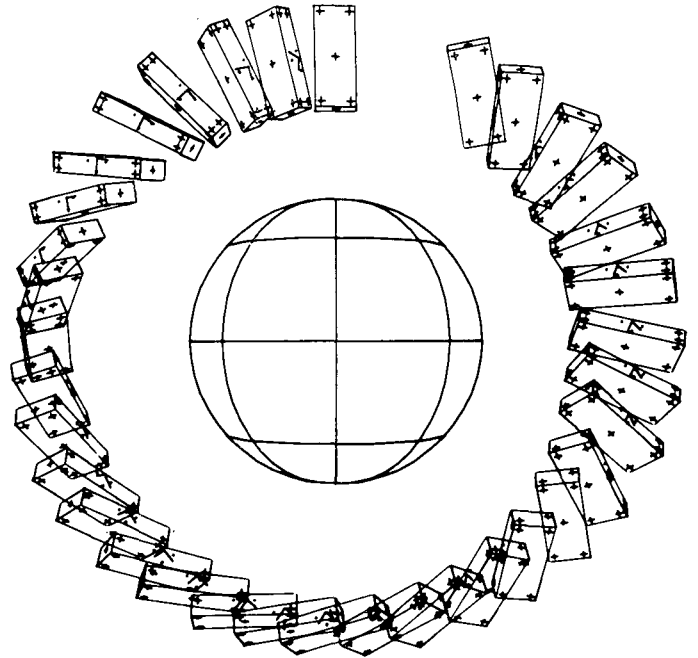


FIG. 2. Computer-made rapid-sequence drawing to illustrate the movie

The second part illustrates the technical application of the movie. It shows how a certain undesirable motion can be made unstable by changing the positions of the gyro stops.

Cost of Movies

The cost depends, of course, on how complicated the movie is. Objects whose shapes are easily described mathematically are simplest to handle. The more complicated the shape, the more expensive the movie. Likewise, an eclipse (i.e. one object partially obscuring another) leads to programming complications in determining the portion of the eclipsed object that is seen, and hence to expense.

The most complicated sequence in the movie shows the satellite orbiting a shaded rotating earth. To compute the drawings for one minute of movie at 16 frames per second, the cost was about eight minutes of IBM 7090 time. In the sequences showing the satellite in a local reference frame, without a rotating earth, the 7090 time was roughly three times the film time. Ancillary costs, microfilm printer, processing, etc., were roughly one-quarter of the 7090 costs.

It might be mentioned that for technical applications, slower projection speeds (say eight or four frames per second as can be obtained on a movie editor) are often satisfactory. Thus, one may use an editor to "see" the motion in a form sufficient for making technical decisions. The cost of this will generally be considerably smaller than that necessary to make a movie to be shown on standard movie projectors.